What is Mathematics?

By Scott Palmer

Last night, I was asked to expatiate on the question "What is mathematics?" I didn't have a lot of time to compose my thoughts, but here they are.

Mathematics is the study of the fundamental structure of reality, together with a set of techniques that enable us to study it.

Although it has many practical applications, its main value is in helping us to understand our world, as well as to show us a powerful, beautiful, and inspiring vision of Truth itself: not of this or that particular truth, but Truth in the Platonic sense, transcendent and all-encompassing.

Only partly in jest, the English philosopher and two-time Nobel laureate Bertrand Russell wrote (in his book *A History of Western Philosophy*) that mathematician Rene Descartes had a simple proof of God's existence:

1. If God does not exist, then mathematics is impossible.

2. But mathematics is delicious.

3. Therefore, God exists.

The sheer delight and deliciousness of mathematics have been the primary motivation of mathematicians throughout the ages. Practical applications, important as they are, have seldom been as significant. The mathematician Leonhard Euler (1707-1783) discovered abstract techniques and concepts that seemed useless in his time, but are now central in understanding particle physics and string theory. The famous British number theorist G.H. Hardy once remarked (in his book *A Mathematician's Apology*) that he had never done any work with a practical application in mind. Hardy added:

> *Any genuine mathematician must feel it is not on these crude achievements that the case for mathematics rests ... In these days of conflict between ancient and modern studies, there must surely be something to be said for a study which did not begin with Pythagoras, and will not end with Einstein, but is the oldest and youngest of all.*

In the 20th century, a few mathematicians denied the transcendent character of mathematical truth. Calling themselves "constructivists," they argued that mathematics was purely a human creation. They contended, therefore, that mathematical truth was limited to what had been, or at least could be, proven by human mathematicians.

But the constructivists' argument could not drown out the clarion call of mathematical inspiration, the irresistible vision of mathematics as truth, beauty, and goodness, that continues to uplift mathematicians and draw them forward.