Axiom of Monotonicity and Axiom of Transfers

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In welfare economics, the axiom of monotonicity and the axiom of transfers are simplifying assumptions used to assess the effect of different income distributions on the general well being (the "social welfare") of a society.

Although they are not true axioms in the logical sense of being self-evident and undeniable propositions (*e.g.,* A=A), the axioms of monotonicity and transfers represent common-sense, plausible generalizations about human nature and human society. For that reason, they are often referred to not as axioms but simply as "principles."

In welfare economics, the two principles are stated in relation to the *social welfare function,* which tries to match different income distributions with different levels of social welfare. Social welfare is conceived, albeit with many complications, as the sum total of individual welfares in the society. Hence, if one person is two units better off while another person is one unit worse off, and everyone else's welfare stays the same, then social welfare increases by one unit.

In essence, the principle of monotonicity states that it is better to have more money (or any other good) than it is to have less of it. Amiel and Cowell (1997) formulate the idea as follows, where x and x' are incomes, X is the income distribution, and W is the welfare function:

The social welfare function is monotonic if, for any x, x' in X, x' > x implies that W(x') > W(x).

The mathematical formulation is not merely a case of academic obfuscation. By stating this common-sense principle in mathematical terms, economists can work out its implications in complex and powerful econometric models.

Simon and Blume (1994) formulate the idea in terms of consumer goods instead of income. Where X is a bundle of consumer goods, $x_1 .. x_n$ are goods in the bundle, and $U(x_1 .. x_n)$ is the individual's welfare derived from a particular bundle, the individual's goal is to buy just the right combination of goods $x_1 .. x_n$ to maximize his/her welfare.

Then for each bundle of goods X, there is some good x_n such that having more of it will increase the individual's welfare -- as Simon and Blume put it, "increasing consumption increases utility." That this principle is *generally* true is evident; that it fails *always* to be true is evident to anyone who has consumed six martinis instead of one.

The principle of transfers states that in a society, greater equality of income is better than greater inequality. In practice, this means that "any transfer of income from a wealthier household to a poorer household will increase social welfare." (USDA, 1998)

Amiel and Cowell formulate the principle as follows, where x and x' are incomes, δ is money transferred from one household to another, *i* is the lower-income household that gets the money, *j* is the higher-income household that gives the money, *k* are all the other households whose income doesn't change, and W is the welfare function:

Where $x'_i = x_i + \delta$, $x'_j = x_j - \delta$, $x'_k = x_k$, then $W(x') >= W(x)$

At its root, this simply applies the idea of marginal utility to the income distribution in a society. Marginal utility states that, other things being equal, a person who has many units of good X will tend to value the next unit of X *less* than would an identical person who had very few units of X.

When applied to money income, this means that a person with a high income (many dollars, euros, etc.) will tend to value an additional unit of money less than a person with a low income (very few dollars, euros, etc.). If a unit of money is *transferred* from the high-income person to the low-income person, the latter gains more welfare than the former loses. As a result, the transfer increases total social welfare.

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